τ, tangential stress, N/m^2; $\bar{\tau}$, dimensionless tangential stress, $[2\tau_w\sqrt{Re}/(\rho U_\infty)]$; φ, reference angle from frontal critical point of cylinder, deg; M, Mach number; Nu_f, Nusselt number $(\alpha d/\lambda)$; Pr, Pr_f, Prandtl number $(\mu_f c_{pf}/\lambda_f)$; Re, Reynolds number $(U_\infty d\rho_f/\mu_f)$; Tu, turbulence of flow $(100\sqrt{u'^2}/U_\infty)$, %. Indices: f, external boundary of boundary layer, $y \geq \delta$; 0, isothermal conditions; ∞, unperturbed flow, m, point of pressure minimum; tr, point of onset of transient flow conditions; t, turbulent layer; Tu, with turbulence of the incoming flow; T, heat-conduction equation.

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FRICTION AND HEAT EXCHANGE IN FLOW OVER A PERMEABLE SURFACE

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UDC 532.542.2

The extremal character of the dependence of friction on suction velocity on a permeable surface immersed in an incompressible liquid flow is established. The suction value corresponding to maximum friction and the limiting heat exchange intensity are calculated.

Interest in the study of transport processes in flow over surfaces made of permeable materials has been stimulated by a number of practical technological applications in both traditional (air and water transport), and new fields of contemporary industry.

Recent studies have shown that the efficiency of drying of sheet and roll materials is increased significantly by thermal processing with a jet draft of heated gas [1]. However, introduction into practice of the progressive techniques realized by this method [2, 3] and development of corresponding methods for calculating equipment parameters [4] demand an ever-increasing understanding of the physical bases of the transport processes involved.

The goal of the present study is to analyze the physical features of thermal and dynamic interaction of an incompressible flow with a permeable surface of a body over which the flow passes. Quantitative data were obtained on hydrodynamics, friction, and heat exchange in the presence of intense surface fluxes of matter. These data can be used independently,

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and may also prove useful in development of more general approaches, based on numerical solution of the complete system of transport equations.

Flow over a permeable surface has a number of unique features which lead even in the stage of problem formulation to significant departures from the classical approaches of boundary-layer theory including draft and suction.

Thus, the intensity of filtration through the wall is determined by the excess gas pressure at its surface. Very often this pressure is determined by the well-known Darcy resistance laws. A consequence of this fact is the variability of the filtration flux along the wall being flowed over: $v_w = v_w(x, p)$ (here and below we use the generally accepted notation). Moreover, it is obvious that for an arbitrary intensity of filtration mass exchange the velocity fields on the body being flowed over (the intensity being determined by the degree of permeability) in the range $0 \le v_w \le u_\infty$ can differ markedly from corresponding values for potential flow over an impermeable body. Finally, as is often found in practice, the main flow may be of a perturbed, turbulized character.

The mathematical formulation of the problem must be based on the complete Navier–Stokes equations, or, for consideration of flow disturbance, on the Reynolds equations for a turbulent flow. The boundary conditions must consider the wall permeability under the action of excess pressure.

The solution of such a problem involves great difficulties, not only, and perhaps, not as much, of a computational, but of a physical character. While these difficulties at present have been successfully overcome in some cases, nevertheless, for example, as regards completing the system of Reynolds equations under conditions of external perturbations, meaningful results are significantly less in number. However, a useful qualitative and quantitative analysis of the problem can be carried out quite simply.

Here we will analyze the hydrodynamic situation which develops along a permeable wall for two asymptotic values of filtration flux: $v_w \ll u_\infty$ and $v_w \sim u_\infty$. If the wall permeability is low, then the filtration velocity will be small. Then as $Re \to \infty$ viscous forces manifest themselves only within the limits of a boundary layer, the thickness of which $\delta_0 \sim \sqrt{x\nu/u_\infty}$ is quite insignificant. It is physically clear that at draft velocities close to the incident flow velocity ($v_w \sim u_\infty$), almost all the material of the latter will pass through the wall, not moving away from it, i.e., $\tau_w \to 0$. As was shown in [5], in this case viscous forces operate within the limits of a layer $\delta_0 \sim \sqrt{\nu R/u_\infty}$.

There is no justification to assume that for any $0 < v_w \le u_\infty$ the order of magnitude of the viscous layer thickness will exceed the asymptotic estimates indicated above.

Thus, in the case of flow over a permeable surface by a liquid at high number Re for any possible filtration velocities it is only in a very small region near the wall that the characteristics of the viscous liquid differ from those of an ideal one.

Gas loss through the wall affects the hydrodynamics and heat exchange with the flow because of change in the boundary conditions for velocity on the wall and the conditions for potential overflow near the wall. We will trace the effect of these factors over the range of variation of filtration velocity indicated above. We shall do this with the example of a circular cylinder with distributed suction of material over its surface.

It can easily be shown that the potential function φ for flow over a cylinder with distributed surface draft of suction in cylindrical coordinates (r, θ) will have the following form:

$$\varphi = u_\infty \cos\theta\left[r + (1 - 2v_w/u_\infty)\frac{R^2}{r}\right] + \frac{R^2}{r} v_w \cos\Theta, \tag{1}$$

with velocity components:

$$u_\Theta = \frac{1}{r}\frac{\partial\varphi}{\partial\Theta} = -u_\infty \sin\Theta\left[1 + \frac{R^2}{r^2}\left(1 - \frac{2v_w}{u_\infty}\right) + \frac{R^2}{r^2}\frac{v_w}{u_\infty}\right],$$
$$v_r = \frac{\partial\varphi}{\partial r} = u_\infty \cos\Theta\left[1 - \frac{R^2}{r^2}\left(1 - \frac{2v_w}{u_\infty}\right) - \frac{R^2}{r^2}\frac{v_w}{u_\infty}\right]. \tag{2}$$

In a coordinate system fixed to the forward critical point (X, Y) [6] these relationships make it possible to write ($v_W = u_W$):

$$\bar{u} = 1 - \frac{1 - \bar{u}_w}{(1 - \bar{X})^2}. \tag{3}$$

From this it is simple to obtain an expression for the velocity gradient in the vicinity of the critical point:

$$d\bar{u}/d\bar{X} = -2(1 - u_w). \tag{4}$$

The coefficient within the parentheses is a correction to the velocity gradient at the wall due to removal or addition of material:

$$d\bar{u}/d\bar{X} = (d\bar{u}/d\bar{X})_0 (1 - \bar{u}_w). \tag{5}$$

Analogous calculations show that this result is also valid for bodies of other form.

Equation (2) can be used to calculate velocity fields for an ideal liquid for any drain (source) intensity on the surface of the body flowed over. Analysis of this expression reveals that in the direct vicinity of the surface the velocity components are quite sensitive to the suction value. This effect is most intense in the vicinity of the forward critical point. Therefore, with no loss of generality we will limit further analysis to this region. An additional advantage of this limitation is the possibility of obtaining quantitative data with simple calculations.

The flow near the critical point belongs to an especially simple class of viscous liquid motions. The Navier–Stokes equations admit transformations which reduce them to a single ordinary nonlinear third-order differential equation. This procedure is widely known [7], and there is obviously no need to repeat its details herein. The key feature of this approach is its representation of the velocity components of the potential flow, which play the role of boundary conditions for the equations of the viscous region:

$$f''' + 2ff'' - f'f' + 1 = 0, \quad \eta = 0, \; f = f_w, \; f' = 0, \quad \eta \to -\infty, \; f' = 1. \tag{6}$$

In this equation and the boundary conditions

$$\eta = [(du/dX)/\nu]^{1/2} X,$$

which defines the velocity components

$$u = -[(du/dX)\nu]^{1/2} f(\eta), \quad v = (du/dX) Y f'(\eta).$$

At a large distance from the wall, at X = $-\infty$, the component u must equal u = (du/dX)X.

Using these relationships, we can calculate the friction for a given suction intensity τ, if the value of the latter on an impermeable surface τ_0 is known together with the corresponding solutions of Eq. (6) f_w'', f_{0w}'':

$$\frac{\tau}{\tau_0} = [(d\bar{u}/d\bar{X})/(d\bar{u}/d\bar{X})_0]^{1,5} \frac{f_w''}{f_{0w}''}. \tag{7}$$

The solutions of Eq. (6) have been studied in great detail [7], and their derivation by numerical methods presents no great difficulty for any $f_w(v_w)$. Analysis of these solutions allows us to approximate with good accuracy the second term on the right side of Eq. (7) by the linear function

$$\frac{f_w''}{f_{0w}''} = 1 + B\bar{u}_w.$$

Then Eq. (7) can be written in its final form:

$$\frac{\tau}{\tau_0} = (1 - \bar{u}_w)^{1,5} (1 + B\bar{u}_w). \tag{8}$$

This expression indicates the extremal character of the behavior of relative friction as a function of suction velocity. Its intensity, corresponding to the maximum value of shear stress, is easily defined:

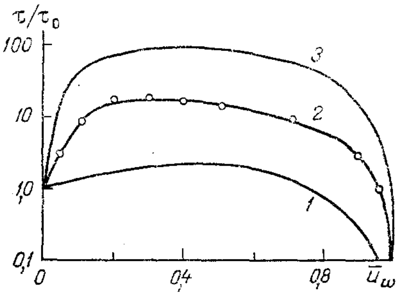

Fig. 1. Effect of suction on surface friction: 1) B = 10; 2) 100; 3) 500 [curves, calculation with Eq. (8); points, numerical calculation for B = 100].

$$(\bar{u}_w)_{\max} = 0.4 - 0.6/B. \tag{9}$$

Curves of relative friction versus suction intensity for various values of the coefficient B are shown in Fig. 1. We will note that in the Re range of practical interest (Re > 10^2), $(\bar{u}_w)_{max}$ differs only insignificantly from its theoretically possible value as $B \to \infty$:

$$(\bar{u}_w)_{\max} = 0.4. \tag{10}$$

As is evident from the curves, calculation by Eq. (8) and exact numerical solution give practically the same result. Thus, in the future, we can use the value of Eq. (10) for engineering estimates of the hydrodynamic situations which develop.

Analysis of Fig. 1 permits quite simple answers to a number of questions regarding the effect of various factors on the change in friction near the permeable surface flowed over: for gas filtration rates through the surface less than $(\bar{u}_w)_{max}$, increase in permeability leads to an increase in friction, while if the permeability of the material is such that $\bar{u}_w > (\bar{u}_w)_{max}$, the friction decreases; for identical suction intensity (materials of equal permeability) the friction increases with increase in Re, and thus, with dimensions of the shell and degradation of flow conditions.

We must also call attention to the magnitude of τ_{max}, which may exceed the friction on an impermeable surface by decades.

We will now consider a problem differing from the preceding one in the presence of a temperature difference between the flow T_∞ and the permeable surface T_w. For definiteness, we assume that $T_\infty > T_w$. We will limit our analysis to the critical point region in the asymptotic case of maximum filtration flow intensity ($u_w = u_\infty$). This approach has the goal of obtaining information on the limiting values of the heat-exchange coefficients.

In this case the energy equation for Pe ≫ 1 and the boundary conditions can be represented in the following form:

$$\rho C_p u_\infty \frac{dT}{dX} = \lambda \frac{d^2T}{dX^2}, \; X = 0, \; T = T_w, \; X \to -\infty, \; T = T_\infty. \tag{11}$$

The solution of Eq. (11) is:

$$T = T_\infty - \exp\left(\frac{u_\infty X}{\varkappa}\right)(T_\infty - T_w). \tag{12}$$

Considering that

$$q_w = \lambda \left(\frac{\partial T}{\partial X}\right)_w \text{ and } \mathrm{St} = \frac{q_w}{\rho u_\infty C_p (T_\infty - T_w)},$$

we obtain St = 1.

Thus, the heat-exchange coefficient increases with increase in permeability of the surface flowed over and in the limit becomes equal to the relative gas flow rate through the surface, it being notable that the thermal conductivity of the flow and the geometry of the body have no effect.

The results obtained and relationships established between friction characteristics and heat exchange may be used to estimate and choose operating regimes for equipment for thermal processing of permeable materials. They also provide a basis for verifying numerical methods for calculation of such processes in a more strict formulation.

NOTATION

u_∞, undisturbed flow velocity; v_w, u_w, surface suction rate; $Re = u_\infty R/\nu$, Reynolds number; τ_w, surface friction stress; $\bar{u} = u/u_\infty$, dimensionless velocity; $\bar{X} = x/R$, dimensionless coordinate; R, characteristic dimension (radius) of body flowed over; f, dimensionless flow function; η, self-similar coordinate; $B \sim \sqrt{Re}/(d\bar{u}/d\bar{X})_0$, velocity gradient parameter; T, temperature, λ, thermal conductivity coefficient; ρ, flow density; $Pe = u_\infty R C p \rho/\lambda$, Peclet number; κ, thermal diffusivity coefficient; q_w, thermal flux surface density.

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KNUDSEN MOLECULAR FLOW IN A CHANNEL WITH A SMALL TEMPERATURE DIFFERENCE AT ITS ENDS

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The thermomolecular pressure difference (TPD) of helium, argon, and krypton is measured in a packet of glass capillaries for temperatures 273 and 293 K at their ends in a 10-100 range of Knudsen numbers.

The temperature difference at the ends of a gas-filled channel results in the occurrence of a mass flow. In a closed system this flow causes the so-called thermomolecular pressure difference.

Elementary kinetic theory, based on the assumption that the density of molecule collisions with the channel wall in the Knudsen mode should not depend on the coordinates, yields the following result [1] for the pressure ratio at the ends of the channel:

$$\frac{P_h}{P_c} = \left(\frac{T_h}{T_c}\right)^\gamma; \ \gamma = 1/2. \quad (1)$$

A strict kinetic solution of the corresponding boundary-value problem with specularly diffuse molecule scattering at the walls allows one to find an expression for the exponent γ of the effect for arbitrary Knudsen numbers [2] in terms of the reduced flux of numbers of particles subjected to the temperature gradient (thermal creep) Q_T and the pressure gradient (Poiseuille flow) Q_P in the form

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